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The article determines the input volumetric capacity of industrial sedimentation centrifuges of different sizes with screw discharge of the sediment.

Sedimentation-type screw centrifuges are used for separating disperse systems with low concentration ( $c < 100 \text{ kg/m}^3$ ) suspended in a large volume of liquid ( $> 0.1 \text{ m}^3/\text{h}$ ) and slowly settling in a gravitational field. The advantage of our installations is that they ensure continuous operation and large capacity.

Already a long time ago it became necessary to devise a substantiated and experimentally confirmed physical model with whose aid it would be possible to calculate the volumetric capacity of a screw centrifuge when the parameters of the centrifuge and of the suspension to be separated are known. At present this capacity is determined in real centrifuges or in centrifuges with similar dimensions with the aid of preliminary experiments. Expenditures on these experiments increase in direct proportion to the dimensions of the centrifuges.

The authors of [1-3] suggested that the capacity determined on a model with the aid of the "equivalent settling area," be converted to an industrial apparatus. The essence of the theory in question is that the process of centrifugal separation consists in gravitational settling proceeding in the apparatus, i.e., the grown field of force is replaced by a large "equivalent settling area."

In the present work we did not make use of Ambler's ideas for the following reason. In continuously operating screw centrifuges the settling particles move radially as well as azimuthally. The introduced suspended matter  $Q_0$  flows in the channel  $h$  toward the outlet  $r_{01}$ . The screw, rotating with the relative number of revolutions  $n_k - n_{sc}$ , entrains the newly settled sediment (see Fig. 1). The place where some particle settles depends on the azimuthal and radial velocities of the stream of liquid. The azimuthal velocity, in turn, depends on the channel section, i.e., on the thickness of the settling sediment, and consequently the processes of settling and of flow have to be regarded as a whole.

The effect of slurry settled in centrifuges on the flow pattern was investigated by Frampton [4] who suggested a semiempirical dependence.

We attempted to determine the steady-state profile of the sediment in centrifuges with screw discharge. We counted on homogeneous distribution of the suspended matter over the inlet section, and we took into account the effect of the changing thickness of the sediment on the flow. With the aid of the model we calculated the volumetric capacities of three centrifuges made in Hungary.

Description of the Model and Basic Assumptions. The main object of investigation was a sedimentation-type screw centrifuge with right thread, counterflow regime, and delayed rotation relative to the drum (Fig. 2). The model was worked out with the following prerequisites and assumptions.

1. The channel has the shape of a rectangular parallelogram (Fig. 2) obtained by developing a screw along the helix. The length of the settling range is

$$x_M = \sqrt{l_0^2 + (2\pi Rn)^2} \approx 2\pi Rn.$$

2. The suspended matter flows azimuthally (in the direction  $x$ ) only, and the velocity of this flow is a function of the cross-sectional area of the given channel, i.e.,

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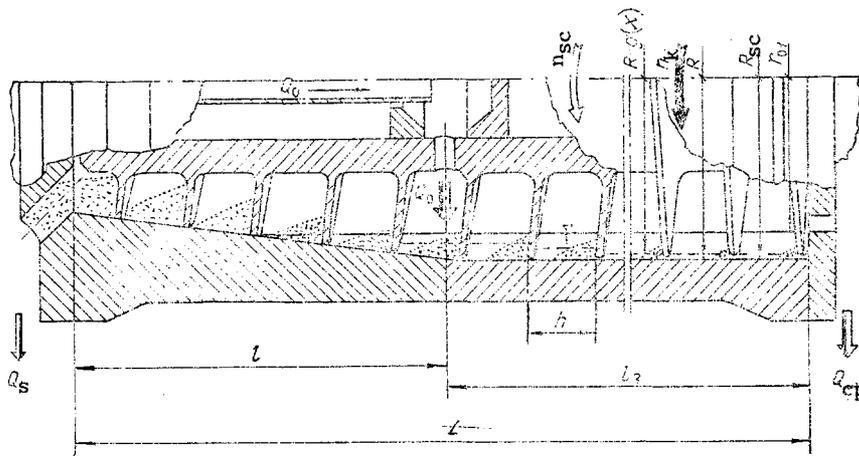


Fig. 1. Diagram and characteristic dimensions of sedimentation centrifuge with screw discharge.

$$v(x) = \frac{Q_0}{h(R - r_{01} - g(x))} \quad (1)$$

3. The centrifugal force acting on the suspended particles is calculated with the aid of the mean angular velocities  $\omega$  determined from the azimuthal velocity of the screw and of the liquid. This angular velocity is approximately equal to the angular velocity of the drum because  $\omega_{sc}/\omega_R \geq 0.98$ .

4. For suspended spheres with radius  $\rho$ , settling in the liquid and rotating with angular velocity  $\omega$ , we neglect the Coriolis force and the force of gravity. Moreover, we disregard the initial acceleration of the particles as well as the radial change of the centrifugal force in the layer of liquid ( $r_{01}/R \approx 0.75$ ); thus,

$$v_0 = \frac{r_{01} + R}{2} \frac{\Delta\gamma}{18\eta} \omega^2 \rho^2 \quad (2)$$

5. The height of the sediment in the model changes only in the azimuthal direction. Instead of the profile of the sediment accumulating in accordance with the dotted line in Fig. 1, we use its mean value (dashed line) in the calculations.

6. At the inlet opening, the particles of suspended matter of all sizes are uniformly distributed over the entire section of the liquid flow (Fig. 3).

Then with the aid of Eqs. (1) and (2) and of the differential equation of material balance we calculate the amount of sediment in the steady state, on the assumption that the sediment has azimuthal distribution across the entire thickness, and that at the given instance the particle distribution is arbitrary.

Description of the Azimuthal Motion of Settling Particles with the Aid of the Differential Equation of the Material Balance. Between the point of settlement of some particle (on the assumption that the design and regime parameters are constant) with radius  $\rho$ , initial coordinate  $r_0$ , and time  $T$  required for settling (determined from (1)), the following correlation exists:

$$R - g(x) = r(r_0, \rho, T).$$

Without concrete specification of the above dependence we can also lucidly represent those values of the variables  $\rho$ ,  $r_0$  for particles which will settle at a certain point  $x$ . The variables  $\rho$ ,  $r_0$ ,  $x$  are represented in the plane  $\rho$ ,  $r_0$  (Fig. 3). We know that  $\rho_m \leq \rho \leq \rho_M$  and  $r_{01} \leq r_0 \leq R - g_0$ . The dashed line of the level shown in Fig. 3 divides the rectangle into two parts, viz., the regions ABC'C and ACD.

Let us first examine the region ABC'C: the point  $x = 0$  (input) corresponds to the straight line  $r_0 = R - g_0$ . Since, according to theory, there is no particle which settles at the point of input, the isohypses do not approach the straight line  $r_0 = R - g_0$ . They either approach the straight line  $r_0 = r_{01}$ , or  $\rho = \rho_M$ .

Particles of maximum size, moving from the surface of the liquid in the region ABC'C, will settle at the point  $x = x'$ , and particles of maximum size moving from the inner

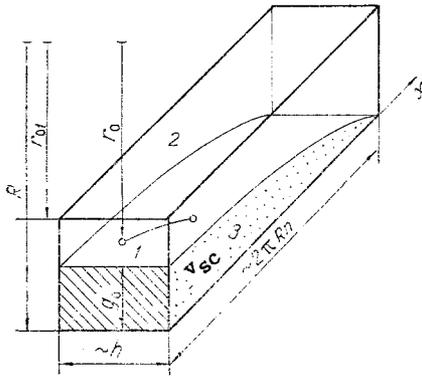


Fig. 2

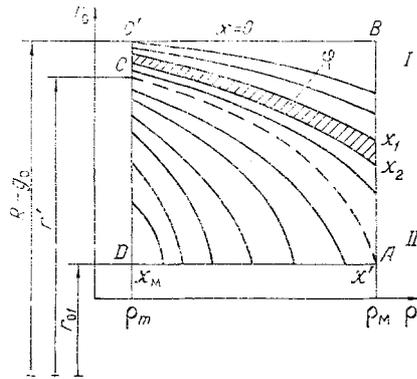


Fig. 3

Fig. 2. Simplified model of settling. Diagram of the developed settling channel with sediment: 1) path of one particle; 2) surface of the liquid; 3) sediment.

Fig. 3. Explanations to the differential equation of balance: I) surface of the sediment; II) liquid surface.

part of the liquid will settle at points  $0 \leq x < x'$ . Particles of minimum size from the region  $r' \leq r_0 \leq R - g_0$  will also settle at points  $0 \leq x < x'$ . This means that if  $\rho_m \leq \rho \leq \rho_M$  and  $r' \leq r_0 \leq R - g_0$ , then  $0 \leq x < x'$ . The end points of the curves  $x = \text{const}$  in the region ACD will be close to the straight lines  $\rho = \rho_m$ ,  $r_0 = r_{01}$ . For  $x < x'$  particles of maximum size, even those coming from the surface of the liquid, will not settle any more, instead, particles with  $\rho < \rho_m$  coming from the region  $r_{01} \leq r_0 \leq r'$  will settle.

Taking the above-said into account, we can write the principal term of the differential equation of balance, but for that it is necessary first to determine the mass of those particles which will settle (see Fig. 3) in the range  $x_1, x_2$ .

Let  $f(\rho) d\rho$  be the number of particles with radii  $\rho$  and  $\rho + d\rho$  in the given unit volume. In that case the concentration is

$$c = \frac{4\pi}{3} \gamma_s \int_{\rho_m}^{\rho_M} \rho^3 f(\rho) d\rho = \frac{4\pi}{3} \gamma_s N \rho^3.$$

The weight of the particles moving into region  $x_1, x_2$  (dashed strip in Fig. 3) within time  $\Delta t$  is

$$\Delta G|_{\Delta x_{1,2}} = \int_{\Phi} \frac{4\pi}{3} \gamma_s \rho^3 f(\rho) d\rho h dr_0 v_x(0) \Delta t, \quad v_x(0) = v_{x0}.$$

If  $\Delta x_{1,2} \rightarrow 0$  and  $\Delta t \rightarrow 0$ , then the integral over the surface may be replaced by the integral over  $\rho$ :

$$dG = \frac{4\pi}{3} \gamma_s h v_x(0) \int_{\rho_m}^{\rho_M} \rho^3 f(\rho) d\rho dr_0 dt.$$

Let us now examine the second part of the equation of the material balance. If in the interval  $x_2 - x_1$  within time  $\Delta t$ , according to assumption 5, the overall height  $g$  increases by  $dg$  as a result of the settlement of slurry (this slurry is carried away by the screw at speed  $v_{sc}$  in the azimuthal direction), then the volume of this slurry is  $hdgv_{sc}\Delta t$ , and its weight will be

$$\tilde{\Delta G}|_{x_{1,2}} = \gamma_i h d g v_{sc} \Delta t, \quad v_{sc} = \frac{\Delta \omega h^2}{\sqrt{h^2 + (2\pi R)^2}}.$$

Since the distribution is steady, we may write:  $dG = \tilde{\Delta G}$ . Using Eq. (1) and unifying the constants, we obtain

$$dg = A \int_{\rho_m}^{\rho_M} \rho^3 f(\rho) d\rho dr_0, \quad (3)$$

where in the counterflow regime sign  $|Q_0| = -\text{sign } |v_{sc}|$ . Thus,

$$A = -\frac{4\pi\gamma_s Q_0}{3\gamma_1 |v_{sc}| h(R - r_{01} - g_0)}, \quad A < 0.$$

Equation (3) changes if  $x > x'$ . Then the upper limit of the integral is the function  $\rho(x)$  which is the solution of the equation  $R - g(x) = r(r_0, \rho, T)$  on condition that  $r_0 = r_{01}$ .

In Eq. (3) we introduce the variable  $x$  instead of  $r_0$ . According to the adopted assumption,

$$r = r_0 + v_0 T, \quad (4)$$

where  $v_0$  is calculated by (2), and

$$r(r_0, \rho, T) = R - g, \quad r_0 + v_0 T = R - g(x), \quad (5)$$

$$T = \int_0^x \frac{dx}{v(x)}, \quad 0 < x \leq x', \quad (6)$$

and if we substitute (6) and (1) into (5), then

$$r_0 = R - g(x) - \frac{v_0 h}{Q} \int_0^x (R - r_{01} - g(x)) dx. \quad (7)$$

Integrating Eq. (3) and using (4), we obtain

$$g(x) - g(0) = A \int_{\rho_m}^{\rho_M} \rho^3 f(\rho) \left[ R - g(x) - \frac{v_0 h}{Q_0} (R - r_{01} - g(x)) \right] d\rho, \quad 0 < x \leq x'.$$

The above integral equation can be transformed into a differential equation:

$$\frac{dg}{dx} = A \int_{\rho_m}^{\rho_M} \rho^3 f(\rho) \left[ -\frac{dg}{dx} - \frac{v_0 h}{Q_0} (R - r_{01} - g(x)) \right] d\rho,$$

whose solution with the initial value  $g(0) = g_0$  we write as follows:

$$R - r_{01} - g(x) = (R - r_{01} - g_0) \exp \left[ -\frac{l_2}{l + l_1} x \right], \quad 0 < x \leq x', \quad (8)$$

where

$$l_1 = A \int_{\rho_m}^{\rho_M} \rho^3 f(\rho) d\rho, \quad l_1 < 0; \quad l_2 = -A \frac{h}{Q_0} \int_{\rho_m}^{\rho_M} \rho^3 f(\rho) v_0 d\rho, \quad l_2 > 0.$$

We obtain the value of  $x'$  from Eq. (7) by substituting  $r_0 = r_{01}$ :

$$R - r_{01} - g(x') = \frac{v_0 h}{Q_0} \int_0^{x'} (R - r_{01} - g(x)) dx.$$

We transform it with the aid of Eq. (8) and solve for  $x'$ :

$$x' = \frac{l + l_1}{l_2} \ln \left| l + \frac{l_2}{l + l_1} \frac{Q_0}{v_0 h} \right|. \quad (9)$$

The solution of Eq. (3) is different when  $x > x'$ . Then the upper integration limit over  $\rho$  is the function  $\rho(x)$  which, after substitution of  $r_0 = r_{01}$  into (5), yields the solution for  $\rho(x)$ .

To determine  $\rho(x)$  from (7), we find  $v_0$ :

$$v_0 = \frac{Q_0}{h} \frac{R - g(x) - r_{01}}{\int_0^x (R - g(x) - r_{01}) dx}.$$

There we must take into account that  $v_0 = \alpha \rho^2$ , where

$$\alpha = \frac{\Delta\gamma}{18\eta} \frac{r_{01} + R}{2} \omega^3.$$

Thus,

$$\rho^2(x) = \frac{Q_0}{h\alpha} \frac{R - g(x) - r_{01}}{\int_0^x (R - g(x) - r_{01}) dx}. \quad (10)$$

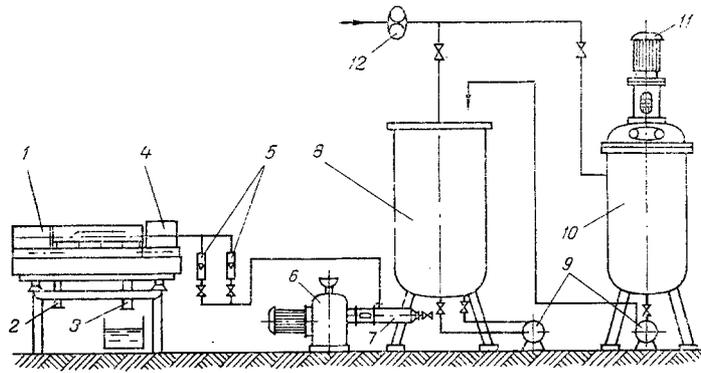


Fig. 4. Diagram of the experimental installation: 1) centrifuge; 2) outlet of purified liquid; 3) outlet of slurry; 4) feed pipe; 5) rotameters; 6) motor; 7) screw pump; 8) 600-liter tank; 9) circulating pump; 10) 250-liter mixing tank; 11) mixing motor; 12) water pump.

It is expedient to differentiate Eq. (10) with respect to  $x$ . After some insignificant transformations we obtain

$$\frac{d\rho}{dx} = - \frac{\frac{dg}{dx} \rho}{2(R-g(x)-r_{01})} - \frac{\alpha \rho^3 h}{2Q_0} \quad (11)$$

Then we express  $dg/dx$  as a function of  $\rho$  and  $x$  in the range  $x > x'$ . We write the derivative of  $g(x)$  with respect to  $x$  from Eq. (3) in the following manner:

$$\frac{dg}{dx} = - \frac{\frac{Ah}{Q_0} \int_{\rho_m}^{\rho(x)} v_0 \rho^3 f(\rho) d\rho [R - g(x) - r_{01}]}{1 + A \int_{\rho_m}^{\rho(x)} \rho^3 f(\rho) d\rho} \quad (12)$$

Equations (11) and (12) form a system of integrodifferential equations determining the functions  $g = g(x)$ ,  $\rho = \rho(x)$ . If we substitute (12) into (11), then instead of Eq. (11) we will have for  $\rho$ :

$$\frac{d\rho}{dx} = \frac{\rho Ah}{2Q_0} \frac{\int_{\rho_m}^{\rho(x)} v_0 \rho^3 f(\rho) d\rho}{1 + A \int_{\rho_m}^{\rho(x)} \rho^3 f(\rho) d\rho} - \frac{\alpha h \rho^3}{2Q_0} \quad (13)$$

With the aid of (13) and a calculation which we do not present here, we obtain

$$\frac{R - g(x) - r_{01}}{R - g(x') - r_{01}} = \exp \left[ \frac{\alpha h}{Q} \int_{x'}^x \rho^2 dx \right], \quad x' < x \leq x_m \quad (14)$$

Now we can already determine the volume of settled slurry relating to the steady regime:

$$H = h \int_0^{2\pi R n} g(x) dx \quad (15)$$

**Experimental Method and Experimental Installation.** In investigations of centrifuge operation, the principal equations describing the processes occurring in the centrifuges are the equations of full material balance and of material balance for solid matter. It follows from our experiments that compiling these balances requires much material and much time if the required accuracy is to be ensured. The cause of this might be that it is extremely complicated to determine the flow rate of the stream of slurry since the material emerges from

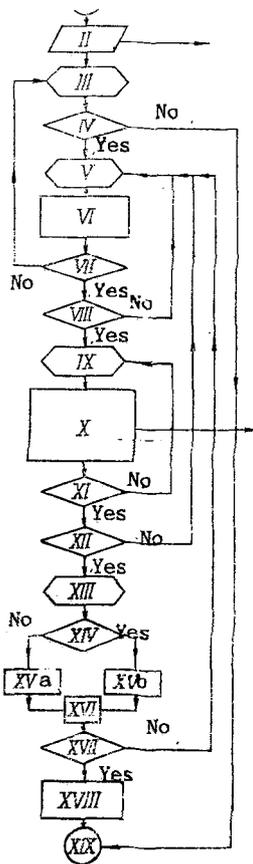


Fig. 5. Block diagram of the calculation: I) start; II) data input  $\rightarrow$  printout; III)  $g_0: -g_0 + \Delta g_0$ ; IV)  $g_0 < (R-r_{o1}), I = \phi$ ; V)  $H-\phi; Q: = Q_0 + \Delta Q_1$ ; VI) calculation of  $g(x')$  by using (8), and  $x'$  by using (9); VII)  $g(x') > (R-r_{o1})/\lambda\phi$ ; VIII)  $x' > \phi x_m$ ; IX)  $-\bar{x} = x' + \Delta x$ ; X) calculation of  $\rho(x = \bar{x})$  by using (13), of  $g(x)$  by using (14), and  $H$  by using (15); XI)  $g(\bar{x}) \leq \phi$ ; XII)  $\bar{x} > x_m$ ; XIII)  $I = -I \cdot 1$ ; XIV)  $I = 1$ ; XVa)  $H-H$ ; XVb)  $H = 100\%$ ; XVI)  $P'$ ; XVII)  $P' \leq P$ ; XVIII) printout of results; XIX) end.

the machine in nonuniform batches: a considerable amount of it is retained on the drum walls from which it may become randomly detached. The concentration of slurry is usually tens of times greater than the input concentration, and therefore the determination of the balance for solid matter is ambiguous.

If the object of the investigation is to determine the concentration of the centrifugal product and the power (like in our case), not the slurry, then it suffices to measure the volumetric flow rate at the intake, the concentration of the suspended matter and of the centrifugal product.

We determined the concentration by the method of measuring the content of dry matter. At the same time we carried out two analyses of samples of the substance and carried out the calculations with the mean values. A diagram of the experimental installation is shown in Fig. 4.

Method of Numerical Calculation. The numerical calculation is based on the system of equations (13), (14), and also on Eq. (8) with whose aid we determined the profile of the sediment for different input flow rates and initial heights of the sediment.

Figure 5 presents the block diagram of the calculation. The input data were determined, on the one hand, by the specified design parameters ( $R, r_{o1}, h, n$ ) and regime parameters ( $\omega, \Delta\omega$ ) of the centrifuges, on the other hand by parameters relating to the suspended matter ( $\gamma_1, \gamma_s, \eta, \rho_m, \rho_M, f(\rho), c$ ) (Table 1).

Experiments at our institution are carried out at present mainly with suspensions of ground chalk whose particle size distribution was determined optically. From the differen-

TABLE 1. Summary of Initial Data

| Parameter                                 | Type of centrifuge     |                        |                        |
|---|------------------------|------------------------|------------------------|
|   | OV-36                  | OV-34                  | OV-33                  |
| $Q_{ini}$ , m <sup>3</sup> /sec, liters/h | 1 · 10 <sup>-5</sup>   | 5 · 10 <sup>-5</sup>   | 25 · 10 <sup>-5</sup>  |
| $g_{o\ ini}$ , m                          | 36                     | 180                    | 900                    |
| $K$ , m                                   | 0,01                   | 0,0135                 | 0,012                  |
| $h$ , m                                   | 0,075                  | 0,11                   | 0,158                  |
| $r_{o1}$ , m                              | 0,03                   | 0,044                  | 0,064                  |
| $n$                                       | 0,055                  | 0,082                  | 0,130                  |
| $c$ , kg/m <sup>3</sup>                   | 9                      | 9                      | 9                      |
| $\omega$ , liter/sec                      | ~23                    | ~20                    | ~20                    |
| $\Delta\omega$ , l/sec                    | 600                    | 400                    | 370                    |
| $\rho_M$ , m                              | 3                      | 2                      | 3,7                    |
| $\rho_m$ , m                              | 2,4 · 10 <sup>-6</sup> | 2,4 · 10 <sup>-6</sup> | 2,4 · 10 <sup>-6</sup> |
| $\eta$ , kg/cm                            | 10 <sup>-7</sup>       | 10 <sup>-7</sup>       | 10 <sup>-7</sup>       |
| $\gamma_s$ , kg/m <sup>3</sup>            | 10 <sup>-3</sup>       | 10 <sup>-3</sup>       | 10 <sup>-3</sup>       |
|   | 2650                   | 2650                   | 2650                   |

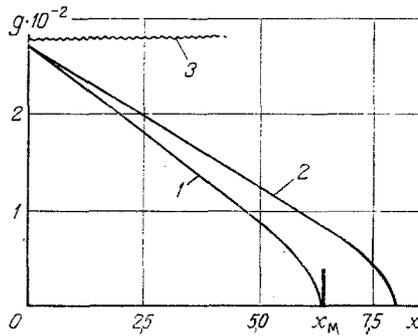


Fig. 6

Fig. 6. Profiles of slurry forming in a centrifuge type OV-34 with different efficiencies ( $\eta P$ ) of distribution (the operational parameters are presented in Table 1): 1)  $P = 100\%$ ; 2)  $92\%$ ; 3) liquid surface.  $g \cdot 10^{-2}$ , m;  $x$ , m.

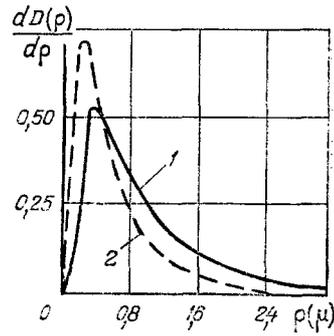


Fig. 7

Fig. 7. Differential size distribution of settling particles: 1) ground chalk; 2) material with finer grain size.

tial size distribution we calculated the mean particle size. The function of the particle size distribution  $f(\rho)d\rho$  was determined from the concentration:

$$f(\rho_i) \Delta\rho = \frac{c_i}{\frac{4\pi}{3} \gamma_s \rho_i^3} \int_{\rho_m}^{\rho_M} f(\rho) d\rho \approx \sum_{i=1}^k f(\rho_i) \Delta\rho,$$

where  $c_i = e_i^1$ ;  $e_i^1$  is the measured parameter of differential volume distribution in the region  $\rho_i$ ,  $\rho_i + \Delta\rho$ .

When we know  $e_i$  and  $e_i^1$  that were measured directly, we can calculate the mean particle size, and also the number of particles per unit volume.

In preparing the computer program, difficulties arose because the profile of the sediment does not depend only on the geometric and regime parameters of the centrifuge and on the parameters of the suspension but also on the initial height of the slurry  $g_0$  and the input flow rate  $Q_0$  in accordance with the system of equations (13), (14) and Eq. (8). The height of the slurry  $g_0$  characterizing the point  $x = 0$  in steady regime is calculated solely from the transient regime, but such a calculation would complicate our model. For the value of  $g_0$  we therefore obtained an estimate on the basis of experimental data, and specifically for suspended ground chalk with a concentration of 20–25 kg/m<sup>3</sup> in a screw centrifuge  $0.5 (R-r_{o1}) \leq g_0 \leq 0 \leq 0.95 (R-r_{o1})$ .

The lower limit is determined by the effectiveness of operation, and the upper limit by the plugging of the conical part. We therefore took the initial value in all cases equal to  $g_0 = 0.5 (R - r_{o1})$  and the step  $\Delta g = (R - r_{o1}) 10^{-4}$  (Fig. 5, III). Proceeding from the initial thickness of the slurry, we increased the input flow rate (Fig. 5, III) until the limit par-

TABLE 2. Effect of Maximum Particle Sizes on the Input Volumetric Capacity

| Type of centrifuge | $\frac{c_{cp} Q_{cp}}{c_0 Q_0}, \%$ | $Q, m^3/h$      |                   |
|--------------------|-------------------------------------|-----------------|-------------------|
|                    |                                     | $\rho_M = 2\mu$ | $\rho_M = 2.4\mu$ |
| OV-36              | 99                                  | 0,30            | 0,35              |
|                    | 94                                  | 0,41            | 0,55              |
|                    | 90                                  | 0,50            | 0,65              |
|                    | 85                                  | 0,60            | 0,75              |
| OV-34              | 99                                  | 0,55            | 0,85              |
|                    | 92                                  | 0,70            | 1,05              |
|                    | 88                                  | 0,90            | 1,20              |

ticle reached the outlet opening (Fig. 5, XII). By integration of Eq. (15) we obtained the retentiveness of the slurry ( $H_{100\%}$ ). Then we continued the calculation of  $Q_0$  until the ratio between the "retentiveness"  $H$  of the slurry and  $H_{100\%}$  ( $H$  is the volume of slurry remaining in a centrifuge that operates with the given efficiency) attained the required value  $P'$  (Fig. 5, XVI). The intrinsic error of the calculation was found to be less than 10%. This was obtained because the effectiveness of separation was calculated approximately as  $P = H_{100\%}/H$  instead of by the exact expression after [5]:  $\eta = c_s Q_s / c_0 Q_0$ .

Results of the Numerical Calculation. Figure 6 shows the profiles of the sediment in a sedimentation type centrifuge with screw discharge, Hungarian made, medium size (type OV-34), calculated on the basis of the physical model. On the side of the discharge ( $x' > x$ ) the height of the slurry decreases abruptly because in the region in question all particles with maximum size have already settled ( $\rho_m < \rho < \rho(x)$ ). Thus, when approaching  $x_M$ , the profile of slurry is formed by ever smaller particles. It is interesting to note that the initial height of the slurry is close to the liquid surface because in our model we did not take into account the effect of reverse flow of the introduced suspension in the direct vicinity of the inlet opening.

Figure 7 shows the differential curve of particle size distribution obtained for a suspension of ground chalk (solid line). In the calculation, the flow rate of suspension related to 100% effectiveness of distribution is  $Q_0 = 0.35 m^3/h$ , and for suspension with finer distribution (assumption) the capacity was  $2 m^3/h$ . Thus, when the mean particle size was reduced by 30%, the volumetric flow rate related to the analogous effectiveness of distribution decreased by 40%. In the assumption adopted by us the correct choice of the maximum grain size is very important. To take the given circumstance into account,  $\rho_M$  was reduced from  $2.4 \mu m$  to  $2 \mu m$  (a cut of less than 10%) which from the point of view of the total volume of the particles is much less than 5%. In this case the mean flow rates related to the same effectiveness of distribution decreased on an average by 20-25% (Table 2) because in connection with the change of the distribution curve, the mean particle size also became smaller.

The object of the numerical calculations was to determine by computer calculation the input volumetric capacities of sedimentation type centrifuges with screw discharge produced in Hungary. Preliminary experimental data at our disposal enabled us to determine the reliability of the model. The volumetric capacities of the centrifuges (Table 3) were calculated from parameters presented in Table 1. The differences between the measured and calculated data in the range of effectiveness of distribution 100-99% are large. This may be ascribed to two causes. Firstly, according to the experimental results, the concentration of centrifugal product  $c_{cp}$  is low, and thus its importance is ambiguous (communication by L. Mishkevits). Secondly, settling particles with sizes close to the minimal ones are returned by the inverse turbulent flow, and as a result the effectiveness of distribution is impaired.

In the range of effectiveness of distribution 85-98% the agreement between experimental and theoretical data was good for three sizes of centrifuges (discrepancy less than 15%), which testifies to the correctness of the conclusions.

Discussion of Results. The theoretical investigations of industrial sedimentation type centrifuges concern an insufficiently studied field of industrial chemistry. The processes occurring in these centrifuges have not been studied much, and their treatment in the literature is insufficient [3, 6]. Most of the published works contain only reference data of a practical nature [2, 3, 7], and the examined processes are greatly simplified (as a rule, these authors carry out calculations for a monodisperse system). Although such results have

TABLE 3. Comparison of the Measured and Calculated Input Volumetric Capacities

| Type of centrifuge | $\frac{c_p Q}{c_0 Q_0}, \%$ | $Q, m^3/h$ |                        |
|--------------------|-----------------------------|------------|------------------------|
|                    |                             | measured   | calc. ( $\rho_M=2,4$ ) |
| OV-36              | 99                          | 0,20*      | 0,35                   |
|                    | 94                          | 0,435      | 0,55                   |
|                    | 90                          | 0,650      | 0,65                   |
|                    | 85                          | 0,800      | 0,75                   |
| OV-34              | 99                          | 1,50*      | 0,85                   |
|                    | 92                          | 1,00       | 1,05                   |
|                    | 88                          | 1,30       | 1,20                   |
| OV-38              | 99                          | —          | 1,90                   |
|                    | 91                          | 2,05       | 2,30                   |
|                    | 86                          | 2,80       | 2,70                   |

\*Information data.

a certain value for practice (since they describe the very fundamental trends of these processes), they do not eliminate laborious experimental investigations or they reduce their extent but slightly. In devising the physical model, we assumed that for the industry an error of 10% is admissible because in experiments a greater accuracy is not attained either, e.g., the calculated and measured values of the magnitudes in the case of suspension of ground chalk for three centrifuges coincided with an error of 15% (Table 3). Our calculations confirm that in the case of specified effectiveness of distribution, the input volumetric capacity (analogously to the experiment) is very sensitive to changes of various parameters and to the particle size distribution (Table 3). The above results show that the accuracy and reliability of granulometric measurements have to be improved. It is interesting to compare the volumetric capacities calculated by using particle distribution, the particle sizes having been measured by different methods (optically, by sedimentation, etc.). In the physical model suggested by us we take into account the flow of "plugs," the flow velocity depending on the cross section of the stream (Eq. 1), and the radial motion of the particles was calculated in accordance with settling after Stokes [7]. We also took the particle size distribution of the suspension into account. The results of the calculation can be obtained only with a computer. However, the expense and time needed for such a calculation are negligibly small compared with the expense and time required for preliminary experiments.

The phenomena that we neglected in describing the process in question may be divided into two groups: 1) refinements not concerning the essence of the physical model and of the calculation, 2) changes concerning the essence of deriving the physical model.

To the first group belongs the assumption connected with the flow pattern of the suspension. Our model ensures the possibility of replacing the flow of the "plug" by another flow pattern for calculating the volumetric capacity of a centrifuge. In comparing the above data with the experiment, we can identify some of them by correlation calculations. However, for that the accuracy of the measurements has to be improved. Among the second group we might class the assumption connected with laminar flow; this is a matter of principle in the choice of sedimentation type centrifuges with screw discharge because in successfully designed centrifuges turbulences are reduced to a minimum because of the resuspending effect. Nevertheless, in practice flow is not always laminar, especially in centrifuges characterized by low effectiveness of distribution and high flow rate. It had already been pointed out earlier [8] that a condition of similarity in screw centrifuges of different sizes is equality of the numbers  $N_{Re}$  and  $N_{Fr}$ , and also the equality of two geometric simplexes. Although these relationships are not obligatory conditions for our calculation, it is nevertheless expedient to choose those groups of centrifuges in which the flows are similar, and with the aid of the correlations to determine the flow pattern for these groups only.

Finally, we want to point out the influence of the geometric simplex  $l_3/R$ . It can be seen from Table 3 that with 99% effectiveness of distribution, the inverse turbulent swirlings arising at the inlet and outlet openings lead to a considerable reduction of capacity. However, this influence in the range of lower effectiveness of distribution is considerable only in machines characterized by the geometric simplex  $l_3/R \geq 2$ . In other cases this effect leads, as a rule, to negligibly small errors.

For the further development of our model and of the program for the calculation it is indispensable to determine the input volumetric capacity of sedimentation type centrifuges with screw discharge characterized by relatively high degree of distribution (more than 75%) and large geometric simplex  $l_3/R$  (more than 2.5).

In conclusion, we express our gratitude to T. Blikle, R. Horányi, and E. Németh for their aid in the preparation of the present article, and also to L. Mishkievits for making available to us the preliminary experimental data.

#### NOTATION

A, matched parameter of Eq. (3); H, retentive ability,  $m^3$ ;  $H_{100}$ , the same referred to 100% effectiveness,  $m^3$ ;  $l_1, l_2$ , matched parameters in Eq. (8),  $l/m$ ;  $l$ , overall length of the centrifuge,  $m$ ; N, number of particles in a specified unit volume;  $Q_0$ , input volumetric flow rate,  $m^3/sec$ ;  $Q_{cp}$ , volumetric flow rate of centrifugal product,  $m^3/sec$ ;  $Q_s$ , volumetric flow rate of slurry,  $m^3/sec$ ;  $P'$ , attained degree of separation; R, inner diameter of the centrifuge drum,  $m$ ; T, settling time of a particle,  $l/sec$ ;  $e_i$ , number of particles in the range between  $\rho_i$  and  $\rho_i + d\rho$ ;  $e'_i$ , the same but for bulk distribution; c, concentration of the input suspension,  $kg/m^3$ ;  $c_i$ , the same but for particles with sizes in the range between  $\rho_i$  and  $\rho_i + \Delta\rho_i$ ,  $kg/m^3$ ;  $c_{cp}$ , concentration of centrifugal product,  $kg/m^3$ ;  $c_s$ , concentration of solid material,  $kg/m^3$ ; dG, term of the differential equation of balance,  $kg$ ;  $\dot{d}G$ , the same from the side of the slurry,  $kg$ ;  $f(\rho)d\rho$ , number of particles with radius in the range between  $\rho$  and  $\rho + d\rho$  in the specified unit volume,  $m^4$ ;  $g_0$ , thickness of the slurry at the point  $x = 0$ ,  $m$ ;  $g(x)$ , local thickness of the slurry,  $m$ ; h, pitch of the helix of the screw,  $m$ ; k, upper summing limit in the calculation of the mean particle size;  $g_m$ , radius of particle with minimum size;  $g_M$ , radius of particle with maximum size,  $m$ ;  $x_M$ , length of the settling region,  $m$ ;  $x'$ , limit of settling of particles with size  $\rho_M$ ,  $m$ ; n, number of helices of the screw at the bottom;  $n_{sc}$ , number of revolutions of the screw,  $l/sec$ ;  $n_k$ , number of revolutions of the centrifuge cylinder,  $l/sec$ ;  $r_{o1}$ , radius of the outlet opening,  $m$ ;  $\gamma_s$ , specific weight of the suspended material,  $kg/m^3$ ;  $\omega$ , mean angular velocity,  $l/sec$ ;  $\omega_k$ , angular velocity of the cylindrical centrifuge drum,  $l/sec$ ;  $\omega_{sc}$ , angular velocity of the screw,  $l/sec$ ;  $\rho$ , hydraulic radius of one particle,  $m$ ;  $\eta$ , viscosity of the suspension,  $kg/m \cdot sec$ ;  $\gamma_1$ , specific weight of the dispersing medium,  $kg/m^3$ ;  $\alpha$ , parameter,  $l/msec$ ;  $v(x)$ , local velocity of the stream of suspension,  $m/sec$ ;  $v_0$ , settling speed after Stokes,  $m/sec$ ;  $v_{sc}$ , speed of slurry removal,  $m/sec$ ;  $l_3$ , length of the cylindrical part of the centrifuge,  $m$ .

#### LITERATURE CITED

1. C. M. Ambler, "The theory of scaling up laboratory data for the sedimentation type centrifuge," *J. Biochem. Microbiol. Tech. Eng.*, 1, No. 2, 185-205 (1959).
2. C. M. Ambler, "Centrifuge selection," *Chem. Eng.*, 78, No. 4, 55-62 (1971).
3. R. B. Bird, W. E. Steward, and E. N. Lightfoot, *Transport Phenomena*, Wiley, New York-London (1960).
4. G. A. Frampton, "Evaluating the performance of industrial centrifuges," *Chem. Proc. Eng.*, 44, No. 8, 402-412 (1963).
5. R. Horányi and J. Németh, "Theoretical investigation of the clarification process in a tube centrifuge," *Acta Chim. Acad. Hung.*, 69, No. 1, 59-75 (1971).
6. R. Horányi and J. Németh, "Investigation of the performance of a clarifier tube centrifuge," *Acta Chim. Acad. Sci. Hung.*, 71, No. 4, 427-444 (1972).
7. R. A. Records, *Chem. Eng.*, 81, 41-45 (1974).
8. V. I. Sokolov, *Centrifuging [in Russian]*, Khimiya, Moscow (1976).